The Haptic Bridge: Towards a Theory for Haptic-Supported Learning

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ABSTRACT
Haptic force feedback systems are unique in their ability to dynamically render physical representations. Although haptic devices have shown promise for supporting learning, prior work mainly describes results of haptic-supported learning without identifying underlying learning mechanisms. To this end, we designed a haptic-supported learning environment and analyzed four students who used it to make connections between two different mathematical representations of sine and cosine: the unit circle, and their graph on the Cartesian plane. We highlight moments where students made connections between the representations, and identify how the haptic feedback supported these moments of insight. We use this evidence in support of a proposed theoretical and design framework for educational haptics. This framework captures four types of haptic representations, and focuses on one – the haptic bridge – that effectively scaffolds sense-making with multiple representations.

ACM Classification Keywords
H.5.2. Information interfaces and presentation: User Interfaces – Theory and Methods, User-Centered Design; K.3.m. Computers and Education: Miscellaneous

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Haptic-Feedback Devices; Dynamic Visualizations; Constructivism; Guided Discovery

INTRODUCTION
Within the last 10 years, new standards in science and mathematics education in several countries have shifted their focus from long lists of content topics to fewer, central ideas in a discipline, with methodological emphasis on active learning practices such as conducting experiments, testing hypotheses, and building engineering devices (e.g., NGSS [27], British Columbia curriculum). These practices have rekindled interest in constructivist approaches to learning, which advocate for sense-making processes and learning environments that include and facilitate active learning activities. Further, technologies designed to create or support constructivist learning experiences can make it easier to scale such experiences to a broader population of students, while expanding possible learning activities. Constructivist learning environments are successful when they lead learners to (and through) moments of cognitive conflict, by providing them with opportunities to restructure their knowledge and achieve deeper understanding, either by interacting with the world, making things, or doing empirical work [9, 19]. Effective constructivist learning environments result from intentional and precise design.

Two particularly effective technologies in the constructivist educator's toolbox are dynamic visualizations (e.g., computer simulations, visualizations, animations) and physical manipulatives. Dynamic visualizations have found widespread use in science and mathematics education due to their low cost, accessibility, and flexibility [30]. Physical manipulatives – like the large, cut-out letters used in Montessori schools – have been used in education for the better part of a century [6]. Well-designed dynamic visualizations and physical manipulatives are both capable of providing students with powerful, effective learning experiences [4, 28].

Haptic feedback devices can function at the intersection of dynamic visualization and physical manipulatives. Most commonly, these devices are used to supply an additional sensory modality (touch) to more traditional computer simulations and dynamic visualizations, creating opportunities for students to touch and feel virtual objects that would normally be manipulated with a mouse and keyboard. The haptic device's sensors send positional information to the computer controller where they are incorporated into the virtual model. Thus, when you push on a virtual wall the virtual wall can "push back".

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As haptic feedback devices become mainstream, they are seeing more use in educational settings. There is little doubt that these haptic experiences are novel, engaging, and potentially useful for education; however, their role in and impact on learning processes involving haptic devices has not been well characterized. In some situations, like motor training simulations for surgery, the learning process seems relatively straightforward. However, learning *abstractions* goes beyond familiarization or sensorimotor memory. Here, there is a critical need for a clear, explicit theory of how haptic feedback impacts learning.

The study reported in this paper was designed to provide insight into how haptic feedback supports learning, and suggest a framework for haptic-supported learning experiences that would inform our future designs. We designed and developed haptic-supported software for learning mathematics called Trigonometry Explorer. We recorded video and audio of a small sample (n=4) of students learning in this environment, and performed two related analyses of the data. The first, a microgenetic analysis of the collected video, provided us with a picture of how the haptic feedback influenced students' abilities to make connections between different mathematical representations. Second, with thematic analysis [21] we developed a novel framework for categorizing haptic learning experiences, wherein haptic representations are broken into two dimensions: the number of corresponding dynamic visualizations, and their resemblance to forces that exist in the physical world.

BACKGROUND

The opportunities explored in this research are newly possible due to the convergence of several technologies and practices with high relevance to education: *dynamic visualizations, physical manipulatives* such as *tangible interfaces*; and *haptic (force feedback) devices.*

Dynamic visualizations are increasingly common in educational settings across disciplines and age ranges. Physics simulations, virtual labs, and mathematical visualizations all aim to promote understanding of concepts using widely available and multi-use hardware (e.g., computers and tablets). Because of their wide availability, low cost, and ability to support open-ended discovery, these types of computational media are a potentially effective way of scaling constructivist learning experiences to a broader population of learners [5].

There is considerable evidence that dynamic visualizations can lead to deeper learning when carefully designed. A particularly effective use is in combining multiple representations in a single interface. Different representations can provide learners with multiple pathways to the core concepts [11, 18], or help manage the "epistemological complexity" of an idea through selective foregrounding and backgrounding [17]. However, there is a danger of overloading the learner's working memory, leaving fewer cognitive resources for sense making, building connections, or constructing mental models [13]. Appropriate supports can address these concerns [3, 28].

Physical manipulatives are three-dimensional, physical objects manipulated with learners' bodies, while dynamic visualizations are two-dimensional, virtual constructs accessed with a mouse and keyboard. Both can be successfully used as building blocks in constructivist learning technologies. In a meta-analysis involving over 7000 students, Carbonneau et. al found consistent, significant results in favor of concrete manipulatives in mathematics education [4].

Theoretical explanations for the effectiveness of physical manipulatives resemble those for dynamic visualizations. When discussing manipulatives, Post writes that "Manipulative materials may now be viewed simply as isomorphic structures that represent... abstract mathematical notions we wish to have children learn... When exposed to a number of seemingly different tasks that are identical in structure, children will tend to abstract the similar elements from their experiences" [20]. Like dynamic visualizations, physical manipulatives are effective in part because they provide learners with multiple perspectives.

In recent years, new learning technologies are emerging at the intersection of dynamic visualizations, tangible interfaces, and physical manipulatives. Haptic feedback devices are one, and have been used as building blocks of many technology-based learning tools and environments: like tangible interfaces, they can support physical manipulation of virtual objects, but also render forces to create the illusion of feeling those objects.

Much of the work on haptic feedback in education is concerned with creating more realistic simulations, including surgical simulators [2], dental simulators [29], and vehicle simulators [1]. All of these aim to create more realistic virtual experiences that better prepare learners to perform tasks in the physical world. Haptic devices have also found use in mathematics and science education, where students can feel mathematical functions and concepts [7, 26], forces generated by objects in virtual laboratories [10], and even molecular forces on the macro scale [22, 23].

Another body of haptic research relates to its use in representing abstractions, as opposed to literal simulations, as laid out in [12]. Recently, a framework describing how users employ multiple interpretive facets in their cognitive sense-making of artificial vibrotactile stimuli underlies navigation of a stimulus library using linked visual representations of those facets [24]. This group previously examined how the addition of force feedback compares to dynamic visualization alone in students' understanding of Newtonian spring forces, finding subtle differences in strategy [14]. Here, we took the next step of looking more deeply into a haptic-facilitated strategy with a qualitative approach.

RESEARCH OBJECTIVES

This study is part of an ongoing program to understand the impact of haptics in education, in which we have developed software and hardware (e.g., [14, 15]), and conducted studies in and out of schools. For more nuanced understanding of how haptic feedback impacts learning, we needed:

1. Deeper understanding of how haptic feedback impacts cognition and learning;

2. A framework to inform the design of future haptic learning experiences.

We determined that these research questions required a different methodological approach than those more commonly used in haptics research (e.g., quantitative comparative studies, or pre/post interviews). We used both microgenetic analysis [25] of video collected during the study to address RQ1 and grounded theory [8] to address RQ2. We present the results of these analyses to illustrate the value of this approach when applied to haptics in learning environments.

METHODS

Participants and Approach

We recruited four high-school seniors (two girls, two boys, aged 17 to 19) from a charter school in the San Francisco Bay Area serving one of the lowest-income communities in California, USA. These students had previously studied trigonometry, but lacked an understanding of the relationship between the unit circle and the graph of sine/cosine on the Cartesian plane. None of them had seen or used a haptic device before the study.

Students worked in pairs to promote discussion and collaboration. We chose to focus on one particular pair (referred to as Manuel and Juan) in our qualitative analysis. While both pairs informed our findings, we focused analysis and reporting on one pair of students for clearer illustration.

This single case study offered insight into the nuances of how haptic feedback and learning can interact, with no attempt to be to be broadly representative. With a qualitative approach, we aimed to capture and analyze the full process of learning with a haptic-supported dynamic visualization, and to use this description and analysis to characterize, design, and assess a haptic learning experience.

Data Collection and Analysis

We collected three primary sources of data. We captured (1) students' interactions with the software by recording screen and webcam video from the interaction computer, (2) audio and video of student interactions with the haptic device and one another with a tripod-mounted camera; and (3) student drawings of the graphs of sine and cosine (Study Phases 2-3).

Our analysis had two components. First, microgenetic analysis [25] of video collected during haptic-supported lessons on trigonometry provided us with key insights into the unique ways that haptic feedback can support students' understanding. Secondly, we employed grounded theory [8] to identify a framework to encompass our observations.

Technological Materials

Haptic Feedback Device: Hapkit

Hapkit [16] (Figure 1) is a one-degree-of-freedom (1-DOF) haptic device. It can be programmed to output forces based on the position of the handle, thus rendering 1-DOF virtual environments based on physical concepts, such as springs and dampers. It is designed as an open-source kit, meant to be accessible and assembled by students and educators, with a cost of approximately US $50, using parts obtained through online purchasing and structural components from digital fabrication tools such as laser cutting or 3-D printing. Hapkit's electronics board allows students to program their Hapkits using any computer, without the need for specialized laboratory equipment. Open-source Hapkit designs are available at http://hapkit.stanford.edu.

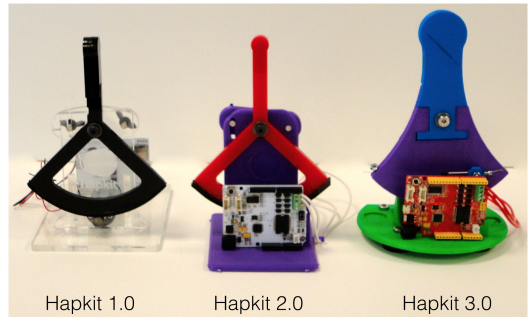

Figure 1. Three versions of low-cost, single-degree-of-freedom, open-source haptic devices for education: Hapkit 1.0 was designed in 2013 and uses laser-cut acrylic structural elements. Hapkit 2.0 was designed in 2014 and a combination of acrylic and 3-D printed plastic structural elements. Hapkit 3.0 was designed in 2015 and uses 3-D printed plastic structural elements.

Software Interface: Trigonometry Explorer

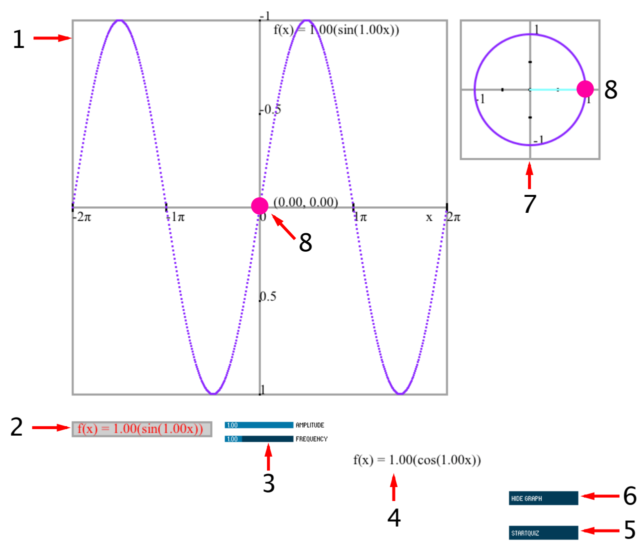

Figure 2. Initial screen of Trigonometry Explorer's Graphical User Interface with labels for each of its 3 components: 1) Graph Box 2) Function Drop Target 3) Amplitude and Frequency Sliders 4) Draggable Functions 5) Quiz Button 6) Hide Graph Button 7) Unit Circle Box 8) User Marker

Trigonometry Explorer (Figures 2, 4) illustrates the relationship between the graph of the trigonometric functions of sine(x) and cosine(x) and the unit circle. The program has two parts: the graphical user interface (GUI), programmed in Processing to run on any personal computer, and the Arduino-based software on the Hapkit Board, which renders haptic feedback and sends position data to the computer. Figure 2 shows Trigonometry Explorer's graphical user interface (GUI)'s eight components:

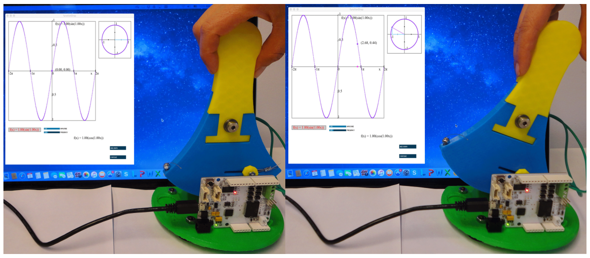

Figure 3. Moving Hapkit from side to side moves the User Marker along the graph and the unit circle in Trigonometry.

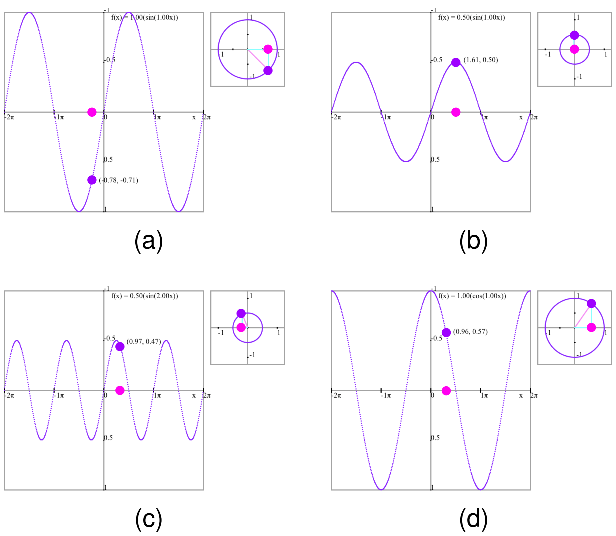

Figure 4. Four different use cases for Trigonometry Explorer. (a) Sine Function selected with Amplitude 1 and Hapkit at -0.78 rad position. (b) Sine Function selected with amplitude 0.5 and Hapkit at 1.61 rad position. (c) Sine Function selected with amplitude 0.5, frequency of 2 and Hapkit at 0.47 rad position. (d) Cosine Function selected with amplitude 1 and Hapkit at 0.96 rad position.

1. *Graph Box:* Shows graph of the function chosen by the user, and the point in the graph the user is currently on using the User Marker.
2. *Function Drop Box:* The user "drops" the function he or she wants displayed in the Graph Box into this box.
3. *Amplitude and Frequency Sliders:* Allow the user to change the amplitude and frequency of the graph chosen via the Function Drop Box.
4. *Functions:* The functions the user can choose to display and feel.
5. *Start Quiz Button:* The software supports a "quiz" modality that was not used for this work.
6. *Hide Graph Button:* Hide the graph from the user. The user can still explore the chosen graph using Hapkit, but will not see it in the graph box.
7. *Unit Circle Box:* Always displays the unit circle and the value of the graph the user is currently on.
8. *User Marker:* Represents the value of the current graph on the Graph Box and the Unit Circle Box.

A student then uses Hapkit as a joystick to move across a graph of a trigonometric function, and "feel" it. Hapkit translates the position the user is at in the graph to a force which the user can feel using the equations $F = A * \sin(f * x)$ or $F = A * \cos(f * x)$, depending on the function chosen. x is the horizontal position of the user in the graph, f is the frequency of the function, and A is the amplitude of the function. The student can change the function's amplitude and frequency using the GUI and see and feel the effects of those changes, while observing the relationship between the graph and the unit circle (Figure 4). When moving the handle, one feels the forces computed from the equations above as either resisting or assisting the motion. The higher above the x-axis, the more force force pushing the handle to the right. The lower below the x-axis, the more force pushing to the left (Figure 5).

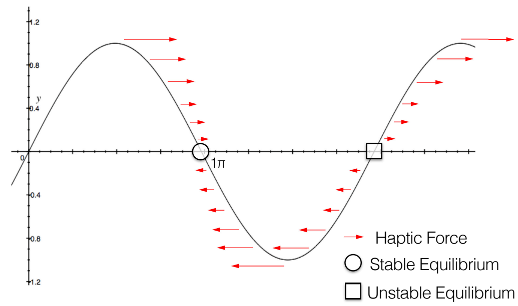

Figure 5. Displacement of the Hapkit handle represents the input to the sine function, and the forces rendered by the Hapkit represent the output.

Procedure

Participants were involved in an hour-long lesson about the relationship between the unit circle and the trigonometric functions sine and cosine. The lesson was designed for students who had studied some trigonometry and were familiar with sine and cosine, but did not fully grasp the relationship between the unit circle and these trigonometric functions. The lesson's learning goal was to help students create the conceptual links necessary to move fluidly between the unit circle and the graph of sine and cosine on the Cartesian plane.

The lesson consisted of four 15-minute phases. The first was a traditional lecture on sine, cosine, and the unit circle. The second and third phases guided students through a haptic-supported, guided-discovery activity: deriving and drawing the graphs of sine and cosine using a dynamic visualization of the unit circle. In the final phase, students guessed the frequency and amplitude of hidden graphs with varying frequencies and amplitudes. We describe these phases in more detail below, along with their intended learning goals and the research questions they were designed to answer.

Phase 1: The Initial Lecture

The first phase was designed to (re)introduce the students to the trigonometric concepts, representations, and methods used to coordinate between them. It did not directly address any research questions, but ensured that the students were familiar with the concepts in the study.

The instructor (an author) led the student pair through a lesson on the unit circle, sine, and cosine. The instructor lectured with PowerPoint slides, asked the students questions, and occasionally moved to the whiteboard to elaborate on specific concepts. The Hapkit was not used.

To determine students' prior knowledge, the instructor asked them if they had learned about sine and cosine in school, if they remembered the "soh-cah-toa" formula, and to draw angles of 45 and −45 degrees, convert from degrees to radians, and use the unit circle to find the sine or cosine of a given angle. The instructor then explicitly showed them how to use the unit circle to find the sine and cosine of a given angle.

Phase 2: Derive the Graph of Sine

The second and third phases of the study were designed to expose the ways in which the haptic feedback impacted the students' understanding of the connections between the two different trigonometric representations. As the students worked through the tasks in these phases, we were able to observe the impact of the haptic feedback through the students' actions, their discussions with each other, and their questions to the experimenters.

In Phase 2, students were asked to derive and draw the graph of $\sin(x)$ using the Hapkit and the unit-circle software. At this time, the actual graph of $\sin(x)$ was not displayed in the central window. The students were given a piece of paper with the central window's coordinates printed out and derive and draw the graph.

Phase 3: Derive the Graph of Cosine

The third phase was identical to the second, except that students were asked to derive and draw the graph of $\cos(x)$. This was more difficult than deriving and drawing $\sin(x)$, because it required students to translate the horizontal position of the point rotating around the unit circle to the vertical position on the graph.

Phase 4: Determine the Unknown Frequency and Amplitude

The fourth phase of the study served to check robustness of the students' interpretation of the haptic feedback. Since the students had not been taught about frequency and amplitude in the initial lecture, their ability to correctly guess the unknown frequency and amplitude would depend on their ability to correctly interpret the haptic feedback and unit-circle animation.

The instructor increased task difficulty by changing frequency and amplitude. Before this, frequency and amplitude of sine and cosine were both fixed at 1. Now, the unit circle radius changed with amplitude, while the rate at which the point moved around the circumference changed with frequency (Figure 6). Students could not see the adjusted values of frequency and amplitude. Then students were asked to determine the frequency and amplitude by viewing the "unit circle" representation and feeling the forces rendered by Hapkit.

FINDINGS

Our first research objective was to arrive at a deeper understanding of how haptic feedback might impact learning. We present this microgenetic analysis under Theme 1, below. Our second research objective, a framework for organizing haptic learning experiences, is addressed under Theme 2.

We offer these findings to illustrate the kinds of data and analyses that are possible using this methodology. Based on a single case study, they should be considered exploratory; further work must determine whether successful interpretation of the haptic representation scaffolds the process of making connections between multiple representations of a single core concept.

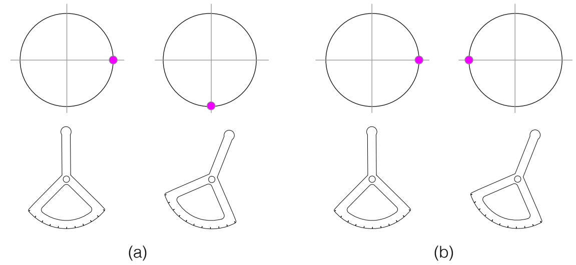

Figure 6. (a) With a frequency of 1Hz, 2cm movement of the Hapkit handle moves the User Marker $\frac{\pi}{2}$ radians. (b) With a frequency of 2Hz, 2cm movement of the Hapkit handle moves the User Marker π radians.

Theme 1: A Microgenetic Analysis of Learning with Haptics

In the following, we present an analysis of the students' utterances, actions, and artifacts from their sessions. Findings are presented in chronological order, and grouped into sections that reflect themes that emerged during the analysis. The first section shows that before working with the Hapkit and Trigonometry Explorer, our featured student pair was unable to make the connection between the unit-circle representation of sine and cosine and the functions' graphs on the Cartesian plane. The second section details how the students learned to make this connection with the support of the haptic representation rendered by the Hapkit and Trigonometry Explorer software. For both students, the haptic representation was initially confusing; however, over the course of the study both students learned to correctly interpret the haptic representation. In the second section, we detail how the students' understanding of the connection between the two different trigonometric representations changed in tandem with their ability to correctly interpret the haptic representation.

Phase 1: Understanding the Pieces but Missing the Connection with Graphics Alone

At the beginning of their session, the students confirmed that they were familiar with sine and cosine, that they understood the relationship between the side lengths of a right triangle and the sine and cosine of its angles ("soh-cah-toa"), and that they had worked with the unit circle. However, in the first phase of the study they struggled to demonstrate this knowledge. For example, when asked to draw a 45-degree angle, one of the students drew a -45-degree angle instead. After a few minutes of help, both students were able to draw both positive and negative angles, but another concept in particular proved more difficult: neither were able to demonstrate how the unit circle could be used to find the sine/cosine of a given angle.

Instructor: *"So we draw a unit circle, and we put an angle there."* Manuel: *"I know one of these has to be sine and one has to be cosine."* Instructor: *"Do you remember which is which?"* Both students: *"No."*

After this exchange, the instructor worked with the students for five minutes to explain how the unit circle could be used to determine the sine and cosine of an angle. During this phase, the instructor explicitly showed the students how to use the unit circle to find the sine and cosine of a given angle. Following this, when the students were given a unit circle and a point of intersection, they were able to correctly determine the sine and cosine of the angle.

This is important because in the second and third phases, the students' task was to use the unit circle to determine the graph of sine and cosine on the Cartesian plane. However, despite the instructor's explicit explanation and their demonstrating their ability to do so in Phase 1, the students were initially unable to use the information from the interactive, dynamic visualization of the unit circle to derive the graph of sine in phase two. At first, the students tried simply remembering the graph of sine, but were unsuccessful. The instructor urged them to use the unit circle representation to re-derive the graph.

Manuel: *"I'm trying to remember which one this is; is it middle-up-middle?"* Instructor: *"You could figure it out right now. You don't even have to remember it."*

Although initially unable to use the unit-circle representation to help them draw the graph of sine, they were eventually able to solve the problem.

Phase 2: Juan Learns to Interpret the Haptic Representation

A primary research goal was to capture the act of student learning with a haptic device – not to determine if haptic feedback could support learning (though we did find evidence of this), but to better understand the *process* of learning with it. Both of the students in our case study utilized the haptic feedback to problem-solve in Phases 2, 3 and 4. Our analysis indicated that haptic feedback helped the students make connections between the different representations for sine and cosine, and that the effectiveness of haptic feedback on learning was tied to the students' ability to interpret it.

At the beginning of the second phase, Juan and Manuel were unclear on how to proceed. After several minutes of re-explaining the concepts from Phase 1, the instructor prompted the students use the Hapkit and software to try and figure out the graph of $\sin(x)$; Juan pulled the Hapkit closer and began to move the handle back and forth. He spent most of Phase 2 systematically manipulating the Hapkit and trying to make sense of what he was feeling and seeing. As he moved it back and forth, he whispered to himself *"middle, up, middle, down."* Each time Juan said a word, the Hapkit visibly rendered a different force. Juan was giving names to the different haptic sensations as he manipulated the handle.

Juan then proposed a hypothesis for how the haptic feedback related to the sine graph: *"So when it's doing this and it gets harder then you're going up higher and higher; then you start getting here and it starts getting easier as it's going down; then you start getting around here again and it starts getting a little harder; at that point you're going up again."*

In this description, Juan refers to "this," "around here," and "at this point." All of these utterances corresponded to a unique Hapkit position. Juan's description of the graph of sine was perfectly synchronized with his movement of the Hapkit. Furthermore, he was correct; the minima and maxima of the graph were located at the two points where the force generated by the Hapkit was highest. The instructor asked Juan to draw the graph. During the one minute and 17 seconds it took Juan to do this, he returned to the Hapkit three times before putting pen to paper. He then drew the graph of sine correctly on the first try.

Phase 3: Manuel's Struggles Lead to a Moment of Insight

At the start of Phase 3, Juan and Manuel traded the Hapkit back and forth to feel the new haptic representation. After a few moments with the Hapkit, Juan announced that he had figured out the graph. While moving the Hapkit, Juan said, *"It starts here at middle, goes up here,... middle again here, I think... down here, and middle here."* As before, Juan's utterances were correctly synchronized with his movements of the Hapkit. When asked by the instructor if the Hapkit helped him figure out the graph, Juan said *"I feel like it helped me figure out exactly what I was looking at."*

Manuel had a harder time than Juan in interpreting the haptic feedback. When Manuel took a turn with the Hapkit, he moved the handle back and forth and exclaimed *"it feels weird... it doesn't stay. [It feels weird] when it changes, [and feels good] at the middle, when it hits the one."* The "one" referred to is labeled 1π in Figure 2, and is the place where the sine wave intersects the x-axis. This is a point of stable equilibrium for the Hapkit; in other words, it is a point where the Hapkit naturally comes to rest. Juan and Manuel agreed that the Hapkit felt "right" when it was in a stable equilibrium, and felt "weird" in other places.

Manuel then moved the handle back and forth while giving verbal labels to the different Hapkit positions. However, unlike Juan, Manuel did not appear to be making the connection between the haptic feedback and the graph of cosine, saying *"I don't know... I know how it looks, cosine, I'm just seeing how can this gonna help it, how's it going to help a person understand how cosine works."*

At this point, the instructor went back to the whiteboard to re-explain how the graph of cosine could be figured out through use of the unit circle. While the explanation was nearly identical to the one presented at the start of the lesson, the result of the explanation couldn't have been more different. After the brief exposition, Manuel had a moment of insight: *"Ohhh, yeah, that's why!"* He then reached for the pen and paper and sketched out his graph of cosine. However, unlike Juan with the graph of sine, Manuel did not return to the Hapkit before putting pen to paper. And also unlike Juan, Manuel failed to get the graph of cosine correct. Although he got the amplitude and point of intersection with the y-axis correct, the frequency was too low (Figure 7, left).

The instructor asked Manuel to try using the Hapkit to improve on his drawing. Manuel took over the Hapkit from Juan, and for the next two minutes, sat quietly using it. Eventually, he grabbed another piece of paper and made a second attempt at drawing the graph of cosine. This drawing was improved

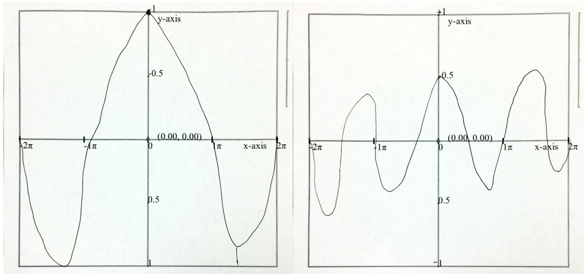

Figure 7. Manuel's first and second attempts at drawing cosine;

(the frequency was much closer to the true graph) but still contained errors (Figure 7, right). When the instructor pointed out what Manuel had gotten wrong, Juan announced that he had seen what was wrong in the drawing but did not reveal it.

Phase 4: Juan and Manuel Work Together to Develop a Complete Understanding

In the fourth and final phase, the instructor secretly varied both the frequency and amplitude of the hidden graph and asked the students to guess them using the Hapkit and Trigonometry Explorer. Instead of drawing the graph each time, the students verbally reported the values of amplitude and frequency to the instructor.

For the first few minutes, the instructor oriented the students to the changes. Both the amplitude and frequency of the function could be changed by moving two sliders on the GUI. As amplitude was decreased, the radius of the unit circle as rendered graphically decreased accordingly. The range of forces rendered by the Hapkit (from minima to maxima) decreased as well (and vice versa). As frequency increased, moving the Hapkit handle resulted in the User Marker moving more rapidly around the circumference of the unit circle (and vice versa).

The instructor asked Juan and Manuel to vary the frequency and amplitude sliders to explore how the unit circle animation and haptic feedback changed. After a few moments, Juan and Manuel guessed at their relationship. Manuel pointed to the amplitude slider and said, *"I know this one is how high it is."* Juan pointed to the frequency slider and said, *"This one is how many loops are in this thing."*

The instructor then set amplitude to 1 and frequency to $\frac{2}{3}$. Both values were hidden from the students; the only way they could determine the new frequency and amplitude was by using the unit circle visualization and the haptic feedback. Juan immediately determined the correct frequency, but was unable to figure out the exact amplitude, saying *"I think it's going to be pretty high... I know the frequency is 3, I just don't know what the amplitude is."* Manuel then reached over and pointed to the unit circle, saying *"You can pretty much tell. Remember that... the x, y would be 1."*

The instructor changed the frequency to $\frac{4}{3}$ and the amplitude to 1. Again, Juan figured out frequency immediately but was unable to determine amplitude, while Manuel was able to determine amplitude by looking at the unit circle. When asked how he was figuring out the frequency, Juan said, *"[I count] the number of times this thing goes in a circle from 0 up to 2π."* Manuel confirmed that he was determining amplitude by looking at the length of the radius of the unit circle.

For the second exercise in a row, Juan determined the correct frequency while Manuel determined the correct amplitude. The instructor then asked Juan to teach Manuel how to determine the frequency, which resulted in the following exchange.

Juan: *"Start here in the middle. Then one full rotation, two, three, four... so four."* Manuel: *"So every time it just clicks, right?"* Juan: *"The number of rotations made in a circle up until 2π."* Manuel: *"You know it's a rotation every time you feel that little... pressure it gives you, whatever you call that."*

Finally, the instructor changed the amplitude and frequency again. This time, the amplitude was set to 0.5 and the frequency was set to $\frac{3}{2}$. Manuel determined the correct amplitude immediately, but got the frequency off by 0.5. The instructor explained that the frequency might not be an integer, and Manuel said *"I know it takes two little stumbles... Ooh! It goes off! Is it 1.5?"* At this point, with little time remaining, the instructor ended the lesson.

In this final phase of the study, Manuel relied on aspects of both the unit-circle representation and the haptic representation when trying to determine the unknown frequency and amplitude. Manuel began this phase unable to determine the unknown frequency. Juan instructed Manuel to *"count the number of times this thing goes in a circle from 0 up to 2π."* Instead of relying only on the graphical, unit-circle representation, Manuel used the haptic feedback to help him identify the phenomenon of interest: *"You know it's a rotation every time you feel that little... pressure it gives you."* As Manuel learned to correctly interpret the haptic representation, he also gained the ability to determine the unknown frequency.

Theme 2: A Proposed Categorization of Haptic Learning Experiences

Here we describe the two major categories we found in our analysis, then use these to identify and describe two important types of haptic representations, each with its own distinct set of learning outcomes. We call these two types of haptic learning experiences "mirror" and "bridge."

Isolated and Connected Haptic Representations

Although haptic representations can be designed to be experienced without a visual component, more often they are designed to accompany dynamic visualizations. A common way of categorizing dynamic visualizations is according to the number of representations present simultaneously. There is evidence that multiple, connected representations are better at helping learners make connections between different aspects of the same concept [11, 18], while individual, isolated representations may be provide other benefits (e.g., lower cognitive load). We extend this categorization to the haptic representations that typically accompany dynamic visualizations, provide examples of existing haptic environments that fit into these categories, and identify the different learning goals that examples in each category best achieve.

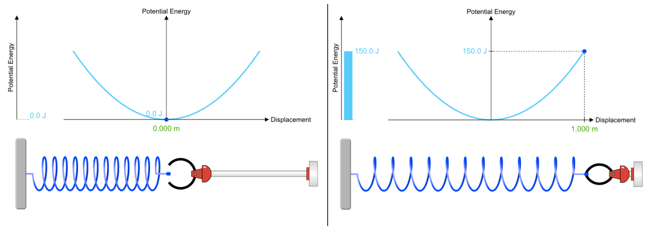

Figure 8. An example simulation that provides a dynamic visualization, but does not contain haptic feedback.

Isolated haptic representations are physical simulations that accompany a single, non-haptic dynamic visualization. As the user manipulates the haptic controller, the visual aspects of the virtual object update accordingly. When a single dynamic visualization is combined with a single haptic representation, often the goal is to create a realistic simulation of interacting with an object in the physical world. For example, isolated haptic representations can render physical simulations of objects including springs, walls, viscous fluids, bodies being operated upon, magnets, and atoms. These types of haptic representations can also provide haptic feedback for virtual objects that are too small or too large to physically manipulate (e.g., atoms, planets) or for objects that are impossible to manipulate directly in the physical world (e.g., mathematical functions, physical quantities like potential energy).

Connected haptic representations are physical simulations that accompany multiple non-haptic, dynamically-linked representations. To help explain these types of representations, we created an example simulation that does not contain haptic feedback (Figure 8). This simulation shows two distinct, dynamic representations: a spring that can be moved using a mouse, and a graph that shows the relationship between spring displacement and potential energy on the Cartesian coordinates. When the user clicks on the spring and drags to the right, both representations respond: the virtual spring stretches and the graph changes to represent the increase in potential energy. These types of dynamic visualizations are designed to help students make links between multiple representations and can lead to deeper conceptual understanding. However, without appropriate pedagogical support, students may struggle to understand the relations between the different representations. This example was modified to contain a connected haptic representation using a haptic device. The forces the user feels when manipulating the haptic device is a connected haptic representation. In our example, the haptic representation connects two visualizations: a spring being stretched and a graph changing in real time.

Isolated and connected haptic representations are similar in that they both provide additional support for students' learning with dynamic visualizations. We hypothesize that each supports a different learning outcome. Because isolated haptic representations combine a dynamic, physical representation with a single dynamic visualization, they seem well-suited to supporting learning about a single, focused concept or single aspect of a complex concept. In contrast, because connected haptic representations combine a dynamic, physical representation with multiple, dynamically-linked visualizations, they seem best suited to helping students make connections between multiple concepts, or between different aspects of complex concepts.

The Semiotics of Haptic Feedback: Iconic and Symbolic Representations

Because the rendered haptic feedback is programmed into the device, the designer must choose the type of haptic representation to accompany the dynamic visualization. In the spring simulation example, a natural choice is to program the haptic feedback device to mimic a spring, so that as the user moves the controller to the right, the user feels a linearly increasing force. Another choice would be to have the haptic device represent the graph of potential energy, so that as the controller was moved to the right, the user feels a quadratically increasing force. We refer to the first type of representation, in which the haptic feedback is designed to mimic a physical object, as an iconic representation, and the second type of representation as a symbolic representation.

When working with an iconic haptic representation, the forces the user feels while manipulating the virtual object are designed to mimic the actual forces that a user would feel when manipulating the corresponding physical object. Symbolic haptic representations make no attempts to mimic physical objects by accurately rendering forces that would be found in the physical world. Instead, these haptic representations are linked to dynamic visualizations in a way that requires interpretation. In our spring example, when the student displaces the virtual spring, she will feel a quadratically increasing force. This haptic representation symbolizes the relationship between displacement and potential energy. If the student correctly interprets the haptic representation, this could aid her in making the conceptual link between displacement and potential energy. However, since there is no way to directly feel the potential energy stored in a displaced spring in the physical world, the student will not be able to draw on analogous experiences in the physical world when trying to correctly interpret the haptic representation. Thus, the student may need additional guidance and support before being able to understand symbolic haptic representations.

Haptic Mirrors and Haptic Bridges

The framework that we have developed allows us to separate haptic learning experiences into four different categories: isolated, iconic haptic representations; isolated, symbolic haptic representations; connected, iconic haptic representations; and connected, symbolic haptic representations. Here, we highlight two of the four categories: haptic mirrors (isolated, iconic representations) and haptic bridges (connected, symbolic representations) (Figure 9). The goal of a haptic mirror is to accurately mimic a physical experience. Examples of haptic mirrors are surgical simulators, vehicle simulators, and physics simulators. Haptic mirrors are useful for simulating situations that are dangerous, expensive, or rarely encountered. Since haptic mirrors are typically used as stand-ins for physical situations, their effectiveness can be measured by comparing students who learn with the haptic mirror to students who learn in the actual physical situation. In most cases, we hypothesize

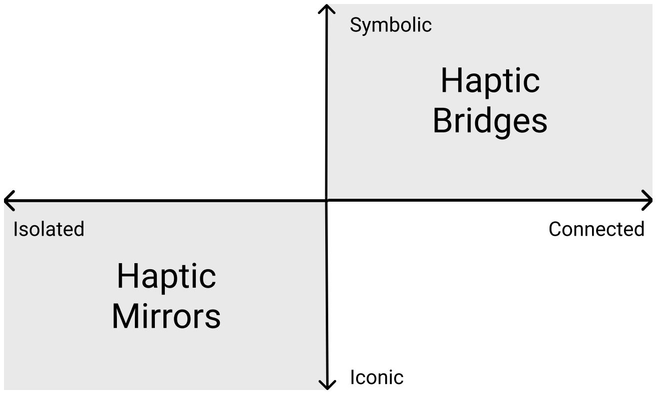

Figure 9. Illustration of our proposed framework. Haptic bridges (e.g., Trigonometry Explorer) occupy the connected & symbolic quadrant, and haptic mirrors (e.g., dental simulators) occupy the isolated & iconic quadrant.

that the more accurately haptic mirrors can reflect the physical world, the better the learning outcomes.

Haptic bridges connect multiple dynamic visualizations with a haptic representation that requires some degree of interpretation. The Trigonometry Explorer is an example of a haptic bridge: one of the dynamic visualizations is the unit circle, and the other is the (hidden) graph of the function on the Cartesian plane. Manipulating the haptic device causes both dynamic visualizations to update, and the haptic representation symbolizes the function's output value. In our analysis, we detailed how interpreting the haptic representation correctly required sustained effort, and how each student's performance was related to his ability to correctly interpret the haptic feedback.

Unlike Haptic Mirrors, the haptic representation of a Haptic Bridge is not necessarily meant to be learned. Once the student has made the connection between the different visual representations, the Haptic Bridge may no longer be needed. This makes it a form of scaffolding; a Haptic Bridge supports students as they learn to make connections between concepts, and once those connections are made it can be removed or forgotten. This is in contrast to the examples of haptic mirrors provided above, where the haptic representation is often meant to be learned.

DISCUSSION

Making sense of the haptic representation required interpretation. That this can be difficult is well known to haptic practitioners; more surprising was that its successful interpretation appeared to be related to the students' ability to solve the problems in each of the phases. Juan learned to correctly interpret the haptic feedback early in the study, and encountered little resistance while working through the remaining problems. In contrast, Manuel failed to interpret the haptic feedback in Phase 2, struggled with the interpretation in Phase 3, and then, with the help of Juan, learned to correctly interpret it in Phase 4. Once Manuel learned to correctly interpret the haptic representation in the final phase—evidenced by his statment that *"you know it's a rotation every time you feel that little. . . pressure it gives you"*—he gained the ability to determine the unknown frequency.

To those who are not haptic designers, the idea of interpreting haptic feedback may seem strange. Many haptic devices are programmed to mimic reality, so that when manipulating a virtual object it feels the same as manipulating a physical object. In these cases, the ability to make sense of the haptic feedback is automatic, bootstrapped by our ability to use our sense of touch to incorporate information about the physical world. However, the haptic feedback incorporated into Trigonometry Explorer is different: the forces rendered by the Hapkit have no real-world analogue. That is, it is impossible to go into the physical world, find a sine wave, and learn how it feels by touching it. In cases like this, the haptic feedback becomes symbolic, which means (in this case) that the students' ability to use the Hapkit to help them imagine the hidden graphs of sine and cosine is dependent on their ability to make meaning of, or interpret, the haptic feedback.

We propose a framework consisting of two dimensions—iconic-symbolic and isolated-connected—for organizing haptic experiences (Figure 9). These dimensions were chosen to organize haptic experiences according to their different learning outcomes. Haptic mirrors are iconic, isolated haptic representations, and are often used for motor training (e.g., learning to drill a tooth). Most examples of educational haptics would be categorized as haptic mirrors. However, when the goal is to improve or achieve conceptual understanding, haptic mirrors are less appropriate. Haptic bridges are connected, symbolic representations that combine a symbolic haptic representation with a dynamic visualization consisting of multiple representations. de Jong et. al write that "It is worthwhile to further investigate the effects of different types of support when offering learners multiple representations. Integrating representations looks promising, but. . . additional support is probably needed to let learners have mindful interaction with the representations" [28]. Although our results are preliminary, there is evidence that the students working with the Trigonometry Explorer—a haptic bridge—were aided in their ability to make sense of the multiple representations by the haptic feedback.

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